CSE560: Homework 5b

Resolution Rule

Question 1

Consider the following KB:
a ∨ b ∨ c
¬a ∨ ¬b

Part a: Enumerate all interpretations, as a truth table, and indicate which ones are models.

Part b: If you apply the resolution rule to two clauses from the knowledge base, getting g, should $KB \models g$?

Part c: For each of the following, (a) indicate which interpretations it is true in, and (b) whether it is true in all models of the KB. Hence, conclude whether the resolution was sound or not.

$$\frac{\begin{array}{c} a \lor b \lor c \\ \neg a \lor \neg b \end{array}}{b \lor \neg b \lor c} \quad \text{Resolve a with } \neg a$$

$$\frac{\begin{array}{c} a \lor b \lor c \\ \neg a \lor \neg b \end{array}}{a \lor \neg a \lor c} \quad \text{Resolve b with } \neg b$$

$$\frac{\begin{array}{c} a \lor b \lor c \\ \neg a \lor \neg b \end{array}}{c} \quad \text{Incorrrectly resolve a with } \neg a \text{ and b with } \neg b$$

Question 2

What are all of the possible results from resolving the following.

$$\frac{\begin{array}{c} a(X,c) \lor b(X,d) \\ \neg b(e,Y) \end{array}}{\quad}$$

$$\frac{\begin{array}{c} a(X,c) \lor b(X,d) \lor f \\ \neg a(e,Z) \lor \neg b(e,Y) \end{array}}{\quad}$$

Which of the above resolutions are examples of unit resolution?

Horne Clauses

Question 3: Horne Clauses

Let KB be the following.

false ← b ∧ c
b ← a
c ← a

Part a: Write the above in conjunctive normal form.

Part b: In a *refutation proof*, to prove g, you take the negation of g and add it to your knowledge base KB giving KB′. If you can prove false from KB′, than g follows from KB.

Consider the query ¬a.

What is its negation, written in conjunctive normal form?

Give KB′, which is your original KB (in conjunctive normal form) along with the negation of your query (also in conjunctive normal form).

Part c: Are all of the clauses in KB′ horne?

Part d: Unit resolution can be viewed as a variation of the bottom up proof procedure where you only do resolutions where one of the resolvents is a unit clause. Also, you do not need to do any pruning with unit resolution.

If KB′ is horne, unit resolution (ur) is guarenteed to find false if false follows from the KB′.
In other words, $KB' \vdash_{ur}$ false if $KB' \models$ false.
If KB′ is not horne, unit resolution is not guarenteed to find false, but you can still use the bottom-up or top-down proof procedure.

Prove ¬a follows from KB using a refutation proof and unit resolution. Show all of your steps.

Question 4: Conversion to Conjunctive Normal Form

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Part a: Write the above in logic. You don't need to use any variables. For instance **mythical** can stand for that the unicorn is mythical. Also, note that immortal is just the same as not mortal.

Part b: Convert clauses into conjunctive normal form.

Is the set of clauses equivalent to horne?
Is the set of clauses equivalent to datalog?

Bottom-Up Proof with Disjunctive & Negative Knowledge

Question 5

Consider the following clauses:

daughter(Daughter,Mother) ← ¬*male(Daughter)* ∧ *mother(Mother,Daughter)*
mother(mary,nancy)
¬*male(nancy)*

Part a: Write each clause in conjunctive normal form.

Part b: Is this datalog? Is this horne? Why or why not?

Parb c: Show, step-by-step, how the consequent set is built for a bottom-up proof. At each step, show what rules you applied, the new consequent set, and what you pruned out and why. From your consequent set, does it follow that daughter(nancy,mary).

Question 6

Consider the following KB:

$poor(X) \leftarrow student(X)$
$student(john) \vee student(tim)$

Part a: Show the resulting consequent set of a bottom-up proof procedure. Show how the set is constructed.